Bayesian Hierarchical Model with Wavelet Transform Coefficients of the ECG in Obstructive Sleep Apnea Screening

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Abstract

Wavelet Transform allows analyzing the properties of a variety of signals, being able to emphasize changes in either time or frequency domain once the appropriate scale is chosen. Since a signal can be expressed in terms of coefficients from wavelet functions, the behavior of this signal could be sparsely represented in these functions, expressing possible properties behind nonstationary signals. Recently, methods based on hierarchical Bayes analysis have been found to be a feasible tool in the approach of physical science and engineering applications. In order to participate in the apnea screening event at the Computers in Cardiology Challenge 2000 and estimate a model that could bring us to an adequate classification between groups we developed the present methodology.

1. Introduction

Apnea is commonly defined as airflow cessation for at least 10 seconds. During obstructive apneas, airflow is impeded by an obstructed upper airway despite continued efforts to breathe. The Obstructive Sleep Apnea Syndrome is a disorder with serious medical, social, and economic consequences. Apnea may cause asphyxia with its effects and provoke arousal during sleep. The present study intends to approach the Obstructive Sleep Apnea Syndrome diagnosis based on the ECG signal using empirical Bayes analysis of coefficients obtained from discrete wavelet transformation to estimate the prior distribution.

Among the advantages of the Bayesian approach, we find the ability to express uncertainty in terms of probability so we can state the posterior probability that either an unit belong to a group or a coefficient is useful to draw certain features of the signal in order to characterize it in a general fashion.

For the particular purpose of the model estimation, discrete wavelet transform (DWT) with several different families of wavelets such as Daubechies and Symmlet was applied to the provided training dataset of ECG signals. The fitting of Bayesian hierarchical model was accomplished using Empirical Bayes Estimation of the hyperparameters in the priors from the distribution of the data. Once the model's parameters were obtained for each group of the training set, the classification was performed based on extracted parameters from the testing dataset. Preliminary results from the Computers in Cardiology Challenge 2000's autoscoring program returned a score of 63.33% using a small set of basis functions. Although this seems to be a low performance, this promising methodology is being evaluated for each set of coefficients from the basis functions, and the assessment of the estimated parameters extracted in order to enhance the classification task.

2. Methods

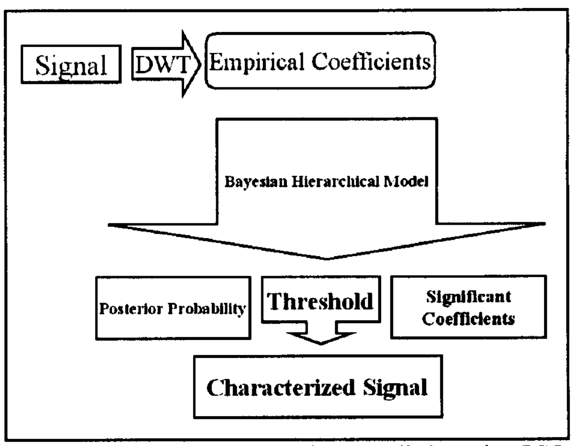

Figure 1. Draws the methodology applied to the ECG database in order to recover the characterized signal.

Signals were provided by Dr. Thomas Penzel of Philipps-University, Marburg, Germany and were available on the Internet for the contest event. The data used in the contest are divided into a learning set and a test set of equal size. Each set consists of 35 recordings, containing a single ECG signal digitized at 100 Hz with

12-bit resolution. Apneas episodes in a minute-by-minute annotation and QRS complexes detection results were available for the training set.

2.1. The discrete wavelet transform

The wavelet transform allows studying properties of the signal on scale and time, revealing dynamics and properties that could be not observed in other kind of analysis.

In this particular study, for all ECG signals from patients of the database, assume in time t:

$$y(t) = f(t) + \sigma z(t)$$

where:

$f(t)$ is the characteristic form of the signal.

$z(t)$ is the random noise. $z \sim N(0,\sigma^2)$, $\forall t$, with known σ^2.

Let $\{\psi_{j,k}\}_{j,k \in Z}$ be an orthogonal wavelet basis of $L^2(R)$, where:

$$\psi_{j,k}(x) = 2^{j/2}\,\psi(2^j x - k)$$

is a dilation at scale j and translation $k/2^j$ of the 'mother' wavelet function ψ. Then the corresponding wavelet coefficient for f will be written as $d_{j,k} = \langle f, \psi_{j,k} \rangle$. In discrete context:

$$\begin{aligned} d_{j,k} &= Wy \\ &= Wf + \sigma Wz \\ &= \theta + \sigma \tilde{z}^{*} \end{aligned}$$

where:

θ is the n-length vector of discrete wavelet coefficients from f.

z^* is an n-length vector of independent and identically distributed $N(0,1)$ observations.

Wavelets are localized in time and frequency, usually providing a good description of a variety of functions in terms of very few coefficients.

Under the assumption of the existence of particular behavior in the ECG signal morphology that could characterize each of the groups, apnea group (A) or normal group (C), segments of signals from group A with apnea episodes and segments of signals from group C were extracted based on available annotations to build the input of the proposed model. Length of the segments were selected to fulfill $j = 13$ in order to take advantage of the apnea annotations and considering the sampling rate of the signal.

Once the input data was constructed, wavelet transform of each signal was applied by way of the MATLAB® library for wavelet analysis, WaveLab v.701[1]. Daubechies and Symmlet bases were considered in the present analysis.

2.2. The Bayesian hierarchical model

Shrinkage of empirical wavelet coefficients is particularly effective when many of these coefficients represent noise rather than signal[2]. Wavelet shrinkage has become a popular method in the signal and image processing areas that seek compression and denoising. Bayesian methods offer coherent data-dependent shrinkage of the transformation coefficients[3].

The present methodology implements wavelets coefficients shrinkage by a mixture model and imposing a particular prior structure onto the model as described by Chipman et al[4] and others[5]. In the prior, the wavelets coefficients of the transformation are mutually independent and each coefficient is a mixture of two normal distributions that estimate that some coefficients of the transformed signals will be zero or close to zero as the following structure:

$$\theta_{j,k} \mid \gamma_{j,k} \sim \gamma_{j,k}\, N(0, c_j^2\, \tau_j^2) + (1 - \gamma_{j,k})\, N(0, \tau_j^2)$$

where:

$$\gamma_{j,k} \sim \mathit{Bernoulli}(p_j)$$

$$p_j = \frac{\{\#coeff\ :\ |d_{j,k}| > \sqrt{2\log(2^j)}\ \sqrt{\sigma + \tau_j}\ \}}{2^j}$$

$$\tau_j = \sigma(coeff.by\ level)$$

$$c_j = \frac{\max|d_j|}{3\tau_i}$$

d_j = are the coefficients in each j level

The multiresolution transformation of the signal provides coefficients that own a distribution behavior on each j level. In this way there is an approach of the value of each coefficient that could arise from a normal distribution with variance $c_j^2\tau_j^2$ or τ_j^2 based on an empirical distribution of the wavelet coefficients in the transformation.

Once the model returns the wavelet coefficients based on their empirical distribution, we seek the posterior distribution of the unobserved coefficients, the distribution of $\theta_{j,k} / d_{j,k}$.

The posterior distribution of θ can be expressed as:

$$F(\theta_{j,k}|d_{j,k}) = F(\theta|d_{j,k}|\gamma_{j,k}=1)\ \Pr(\gamma_{j,k}=1|d_{j,k}) + F(\theta|d_{j,k}|\gamma_{j,k}=0)\ \Pr(\gamma_{j,k}=0|d_{j,k})$$

where:

$$\Pr(\gamma_{j,k}=1 \mid d_{j,k}) = \frac{O_{j,k}}{O_{j,k}+1}$$

with

$$O_j = \frac{p_j\, N(0,\sigma^2 + c^2{}_j\, \tau_j^2)}{(1-p_j)\, N(0,\tau_j^2+\sigma^2)}$$

The model was developed in Borland C++® and analysis were made using MATLAB®. The input signals of the model in either case were based on the apnea minute-by-minute annotations provided by the training set database.

As can be seen in Figure 1, the value of a threshold is required to define the coefficients recovered by the model which the posterior probability must exceed in order to be accepted. In the present study this threshold was explored with values of 0.6, 0.7 and 0.8.

Frequency distribution of returned parameters from the model were analyzed and compared to those obtained from random segments of the testing set in order to achieve classification between groups.

3. Results

Although the goal of this study was to classify the two described groups (A and C), it is important to notice the way the model is able to characterize the input signal based on the coefficients left by the model in the analysis. In Figure 2 can be observed a reconstructed signal by inverse wavelet transform based on a subset of the input coefficients and a threshold of 0.6 of a signal in group A. This analysis was developed using several thresholds in the model and found to be critical between values of 0.6 and 0.7 in the training set and the family of wavelet used in the transformation did not affect this result.

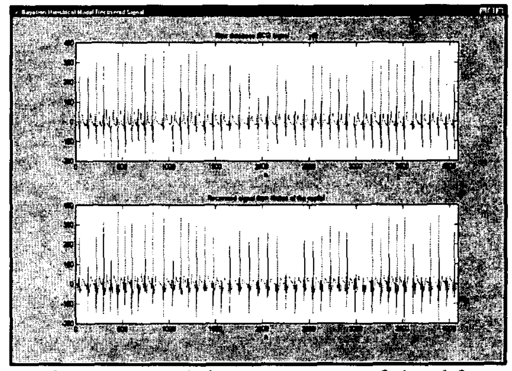

Figure 2. Upper panel shows a segment of signal from the database and the bottom panel is the inverse wavelet transform of the coefficients returned by the model.

In order to classify between groups based on the wavelet coefficients returned by the model, frequency plots were constructed. Figures 3 and 4 belong to group C and the latter two plots belong to the group A.

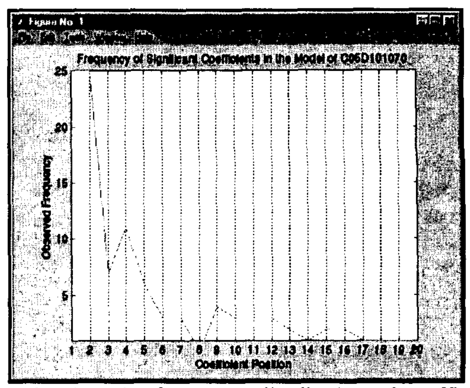

Figure 3. Plots the frequency distribution of coefficients obtained until level 3 from a group C patient.

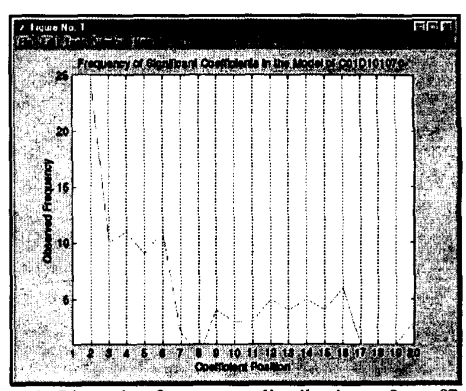

Figure 4. Plots the frequency distribution of coefficients obtained until level 3 from a group C patient.

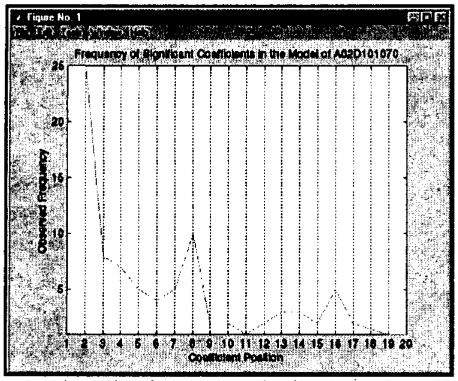

Figure 5. Plots the frequency distribution of coefficients obtained until level 3 from a group A patient.

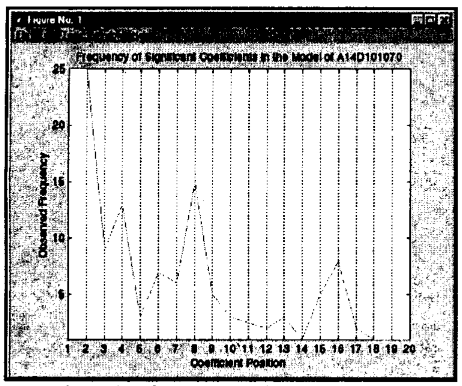

Figure 6. Plots the frequency distribution of coefficients obtained until level 3 from a group A patient.

4. Conclusions

Classification between groups A and C was not appropriately developed based on the frequency distribution of the coefficients returned by the model. The score of 63.3% for classification of the testing set was not enhanced using different wavelet basis but a certain pattern observed in the frequency distribution of the coefficients in the training set was shown. This pattern seems to express differences corresponding to $j = 3$ and is observed that in group A, the frequency distribution of the wavelets coefficients among this level behaves in a different fashion to the distribution in group B.

5. Discussion

The presented model based on Bayesian Hierarchical Modelling shows a very appropriate analysis to characterize the signal, although the performance of the classification task is low, this could be enhanced.

A possible bias could be the fact that in the testing set, as in the training set the signal of either condition is represented, even when in the model this could characterize the signal, there is a mixture of conditions. In the case of training set, based on the minute-by-minute acknowledge of the condition, this feature was possibly recovered from its condition but in the testing set these apneas and normal breathe episodes are mixed and samples of these mixtures are not well represented by the model.

References

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